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THESIS

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ABSTRACT

In video magnetic recording the distortion effects of the head circuits and recording media are basically undefined. No analysis technique has been available in the literature. This paper establishes a method for computing the minimum distortion to be expected due to the video head circuit. Moreover, a new technique for investigating video head circuits and equalization effects is given. The classical FM approach is justified and used. Insight into phase distortion is achieved by use of a "dynamic" rather than "spectral" analysis. Computational techniques are indicated.

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TABLE OF SYMBOLS AND ABBREVIATIONS

Ω = frequency (radians/second)

Ω_i = instantaneous frequency

Ω_c = carrier frequency

$\Delta\Omega$ = frequency deviation

ω = modulating frequency

Ω_{res} = resonant frequency of video head circuit

Q = resonant Q of video head circuit

$Z(j\Omega_i)$ = impedance of the video head circuit at

$A(\Omega)$ = n^{th} order polynomial approximating the amplitude response of the video head impedance

$\phi(\Omega)$ = n^{th} order polynomial approximating the phase response of the video head impedance

FM = frequency modulation

1. Introduction.

In the effort to improve video magnetic tape recording, many limiting obstacles have arisen not only in the optimization of existing circuitry and components, but also in the theoretical understanding of the record/reproduce process. One of the most interesting problems is in seeking a hard-wearing ferrite, machineable to close tolerances and of the desired magnetic properties, for use in the video head assemblies.

Once such a material is had, the effect of its permeability over the frequency range of interest must be evaluated to determine what distortion is induced either directly or indirectly, since the head is resonated to improve bandwidth characteristics.

The purpose of this paper is to show that video band distortion can be analyzed to a first order approximation by resolving the problem into a classical analysis of tuned circuit distortion of an impressed frequency-modulated wave.

Unfortunately, many misconceptions exist in evaluating tuned circuit distortion. The spectral analysis of system response has long been the basis for design. Spectrum Analysers are available at reasonable prices and undoubtedly channel thought around these instruments. They do not, however, yield any insight into phase distortion, a first order effect of distortion to an FM wave. For ease of computation investigators have developed a "dynamic" analysis as opposed to "spectral" such that both the amplitude and phase characteristics of a given circuit may be weighted properly. Early contributors to this "dynamic" technique were Carson and

Fry[1], Van der Pol[2], and Stumpers[3].

The results of this investigation suggest (1) a new way to consider video head distortion, both qualatatively and quantatatively and (2) a means for extending this theory to investigate the effects of video head phase and amplitude equalization.

[1]J. R. Carson and T. C. Fry, Variable-Frequency Electric Circuit Theory, B. S. T. J., 16, pp. 513-540, Oct., 1937

[2]B. Van der Pol, The Fundamental Principles of Frequency Modulation, J. IEE (London), 93, pt. 3, pp. 153-158, May, 1946

[3]F. L. H. M. Stumpers, Distortion of Frequency Modulated Signals in Electrical Networks, Commun. News, 9, pp. 82-92, Apr., 1948

2. Brief description of video recording system.[4]

While there are several video tape recording techniques only that used by Ampex will be considered here. A very high writing speed is attained by recording transversely across the video tape while pulling the tape at a rate dictated by the width of the recorded tracks and the number of these tracks laid down per second.

Four heads are mounted in a circular drum so that their tips protrude very slightly past the periphery of the drum. These heads are precisely aligned to be 90 degrees apart. With a drum diameter of two inches and a rotational speed of, say, 240 revolutions per second, a writing speed of 1500 inches per second is obtained. The video tape is two inches wide which means that each head covers about 120 degrees of arc while in contact with the tape. This allows for ample time for switching input or output signal between two successive heads. The same heads are used for both recording and reproducing. Control and sound tracks are laid down on the edge of the two inch tape with standard audio heads. Conventionally, the two inch tape travels at 15 inches per second giving a five mil spacing between adjacent edges of the recorded tracks, each track being 10 mils wide.

No discussion of the control system will be given except to mention that its purpose is to establish a recorded time reference so that during playback the same time base can be reproduced.

[4] C. E. Ginsburg, Description of the Ampex Videotape Recorder, J. Society of Motion Picture and Television Engineers, 66, No. 4, Apr., 1957

3. The modulation system[5].

Frequencies much lower than 50 cycles per second give a recorded flux change very slowly per unit of tape length with a low resultant output voltage. Increasing the speed of the tape increases the difficulty of reproducing the lower frequencies.

The high frequency end of the recorded spectrum presents an opposite effect to the low frequency end. Here the wavelength on tape must be maintained greater than the gap width of the playback head. This is done in two ways: either by increasing the tape speed or by decreasing the gap width. Usually, both means are employed.

Clearly then, to improve the high frequency end is to sacrifice the low frequency and vice versa. Since wide band video requires signal recording from, roughly, 10 cycles per second to four megacycles per second some processing of the video signal must be performed in order to record and recover the desired bandwidth. For this reason various modulating schemes have been investigated.

Amplitude modulation yielded variation in output level between playback heads. This was due to manufacturing differences between heads. The control system should keep the playback head in the center of the recorded track. Variation from the center of this track resulted, again, in a loss of amplitude. These two problems suggested an instantaneous AGC system. However, circuit considerations obviated this approach.

A frequency modulated system finally evolved from the Ampex

[5] C. E. Anderson, The Modulation System of the Ampex Videotape Recorder, J. SMPTE, 66, pp. 182-184, Apr., 1957

efforts. It violated the basic assumptions of the classical derivations. Namely the modulating frequency approaches the carrier frequency and the frequency deviation is not much smaller than the carrier frequency.

The classical derivation is given in Appendix I. The approximations and the reasons for making them are clearly given. Of note is the final result of the FM wave being expressed as:

$$q = K \operatorname{Cos}(\Omega t + \frac{\Delta F}{f} \operatorname{Sin} wt) \qquad (1)$$

where, q = charge

K = constant

The development of this expression into the Bessel expansion is exact. It yields:

$$\begin{aligned} q = J_0(\beta) \operatorname{Cos} \Omega t \; & - \; J_1(\beta) \; \operatorname{Cos}(\Omega - \omega)t - \operatorname{Cos}(\Omega + \omega)t \\ & + \; J_2(\beta) \; \operatorname{Cos}(\Omega - 2\omega)t + \operatorname{Cos}(\Omega + 2\omega)t \\ & - \; J_3(\beta) \; \operatorname{Cos}(\Omega - 3\omega)t - \operatorname{Cos}(\Omega + 3\omega)t \\ & + \; J_4(\beta) \; \operatorname{Cos}(\Omega - 4\omega)t + \operatorname{Cos}(\Omega + 4\omega)t \\ & \ldots \end{aligned} \qquad (2)$$

From this expression then the usual spectral picture of the FM wave is obtained but in the Ampex system it must be understood what assumptions were made in arriving at equation (2).

In present development and production equipment unusual modulation techniques are being employed. The signal to be recorded is applied to the modulator to frequency modulate the 36 megacycle per second Hartley oscillator. This oscillator circuit uses two voltage sensitive Varicap semi-conductor capacitors in the tuned circuit. Neglecting the non-linearities of the Varicaps, this

generation of an FM wave has been especially treated by Cambi.[6] His development, however, is limited only to the case where either inductance or capacitance of the tank can be varied in a sinusoidal manner. Any application of this derivation to any other means of FM generation has no foundation in mathematical rigor. Happily, it does apply to current development trends at Ampex. (Neglecting Varicap non-linearities as previously mentioned).

However, the frequency at which the FM wave is generated and the deviation and modulating frequencies used must be carefully considered.

Cambi has shown that in this system of FM generation there is a shift of energy to the upper frequency components and a slight shift in mean center frequency. Under the Ampex system, the frequency shift is entirely negligible and the energy shift not detectable as viewed on a Spectrum Analyser.

The trend in the frequency generation is upward. Currently the frequency used is 36 megacycles per second but frequencies to 42 megacycles per second have been used. Higher frequencies are anticipated to avoid such problems as video feed-through of the modulating frequencies. Regardless of the frequency used, a heterodyning system is employed to translate the spectrum to center on the actual carrier frequency recorded on tape. If we can assume a linear translation, then the relation of all side frequencies to the carrier frequency is preserved. Foldover for those spectrum components which would be translated into the negative frequency domain will, in any case, be a problem. However, these foldover

[6] E. Cambi, Trigonometric Components of a Frequency Modulated Wave, Proc. IRE, 36, p. 42-49, Jan., 1948

components are generally over 45 db down from the carrier frequency.

As an upper limit to the Cambi equations for an FM wave, as we get higher in frequency and approach the conventional FM case as developed in Appendix I, it would seem intuitive that the energy distribution would tend to be more symmetric. Indeed, this is the case and even at a carrier frequency of 36 megacycles per second this is so. Table 1 gives comparison values of spectral components for the Cambi and the classical approximations at a carrier frequency of 36 megacycles per second, deviation of .5 megacycles per second and modulating frequencies of one, two, three and four megacycles per second.

The inherent advantage of using a closed expression for the FM wave as given by the classical derivation as opposed to the more rigorous Cambi derivation, which cannot be expressed in a closed form, is immediate. To realize this it is first necessary to develop the background for the dynamic analysis. However, we shall defer this treatment until later. Suffice now to conclude that the classical derivation is sufficiently close to the more exact Cambi solution to justify usage of the former.

Table 1

Comparison of Spectral Component Amplitude (db)

	F = 36 mcs				F = .5 mcs			
Spectral	f = 1 mcs		f = 2 mcs		f = 3 mcs		f = 4 mcs	
Component	Classic	Cambi	Classic	Cambi	Classic	Cambi	Classic	Cambi
-3	-51.3	-54.0	-69.0	-74.4	-79.1	-87.6	-84.4	-101.8
-2	-29.7	-31.3	-41.9	-44.4	-49.2	-52.7	-53.9	-59.0
-1	-11.8	-12.4	-18.0	-18.8	-21.6	-22.7	-24.0	-25.6
0	0	0	0	0	0	0	0	0
1	-11.8	-11.7	-18.0	-17.3	-21.6	-20.5	-24.0	-22.7
2	-29.7	-29.1	-41.9	-40.1	-49.2	-46.2	-53.9	-50.2
3	-51.3	-49.6	-69.0	-65.7	-79.1	-74.4	-84.4	-80.0

4. General considerations in recording/reproducing.

To this author's knowledge there has never been a complete mathematical model of the record/reproducing process in magnetic recording. The multiplicity of limiting factors and system nonlinearities both in the record/reproduce head assemblies and in the recording media have been sufficiently complex and interacting to defy a complete description.

Contributing works by Begun[7]; Wallace[8]; Westmijze[9]; Hoagland [10]; Daniel, Axon and Frost[11]; and others have successively updated the state of the art in this difficult field.

From a qualitative treatment of tape recording it is seen that the distribution of magnetization on the surface of the recording media resembles in space the recording current in time. Since the gap flux and recording current are very large values, it seems intuitive that the magnetization on the tape would follow directly the current in the head coil. While such factors as tape thickness and self demagnetization in the tape cause a departure from this intuitive concept it is at least good for a first order approximation.

[7] S. J. Begun, Magnetic Recording, Murray Hill Books, Inc., 1949

[8] R. L. Wallace, Reproduction of Magnetically Recorded Signals, B. S. T. J., 30, pp. 1145-1173, Oct., 1951

[9] W. K. Westmijze, Studies on Magnetic Recording, Philips Research Reports, 8, 1953

[10] A. S. Hoagland, Magnetic Drum Recording of Digital Data, AIEE Trans., 73, pp. 381-385, Sept., 1954

[11] E. D. Daniel, P. E. Axon, W. T. Frost, A Survey of Factors Limiting the Performance of Magnetic Recording Systems, Proc. IEE, 104, Part B, pp. 158-168, March, 1957

A blurring effect is to be expected due to fringing of the flux in the head gap. The extent of this fringing will set an upper limit of resolution for any given recording situation. (e.g., head parameters, recording media thickness, etc.,). The magnitude of bias current, uniformity of the recording field (and, in turn, the recording gap width) and the rate of extinction of the recording field as an element of tape leaves the recording head all are factors affecting this upper limit of resolution.

The more difficult aspect of a head model comes from playback considerations. The fundamental losses encountered in playback are adequately covered in the references previously cited and are given here with no attempt at derivation.

1. Self demagnetization
2. Eddy current and other losses in the head
3. Gap loss due to the finite scanning slit in the reproduce head, given as

$$20 \log_{10}\left[\frac{\pi g/\lambda}{\sin(\pi g/\lambda)}\right] \text{ db}$$

where, g = effective gap width

λ = recorded wavelength

4. Spacing loss due to imperfect magnetic contact between the reproducing head and the recording media. This is given as

$$20 \log_{10}\left[e^{-\frac{2\pi d}{\lambda}}\right] \text{ db}$$

where, d = spacing between head and tape

5. Thickness loss, given as

$$20 \log_{10}\left[\frac{2\pi \delta/\lambda}{1 - e^{-\frac{2\pi\delta}{\lambda}}}\right] \quad \text{db}$$

where, δ = thickness of recording media

Other considerations in playback also exist[12] but these five items represent the major considerations. With the exception of eddy current losses, all represent losses in amplitude of the reproduced signal.

The sensitivity of a reproduced head is proportional to the ratio of front-gap reluctance to total reluctance.[13] While the usual head has a well defined gap at the front it is customarily achieved by manufacturing the head core in two halves and clamping them together. In the Ampex video head, a refinement is used in that within the front gap of the main ferrite core, an Alfenol core is inserted. W. T. Frost of Ampex Corporation [13] has shown that over the frequency range four to 10 megacycles per second, the head may be considered to consist entirely of an Alfenol core of length determined by the effective length of Alfenol within the ferrite core gap. The purpose of the ferrite is to minimize the effective length of Alfenol, thereby maintaining a higher sensitivity than would otherwise be possible.

He has further shown that over this frequency range the sensitivity contributes a constant 45 degree phase shift. His treatment in general has resolved the problem of eddy current, hysterisis and residual losses in terms of complex permeability. Happily then, this factor cannot contribute to an unpredictable phase

[12] E. D. Daniel, et al., loc. cit.

[13] E. D. Daniel, et al., ibid.

distortion of the FM wave. Therefore, no further consideration to head sensitivity shall be given in this paper.

5. Mathematical model of head.

Reviewing the conclusions already reached, we see that (1) the classical FM wave approach may be used over the more difficult Cambi expression, (2) the major consideration in recording/reproducing is in playback, so consideration of the latter function only will yield, at least, first order effects, and (3) the playback problem is limited to amplitude loss as a function of frequency.

Limiting action is employed in the Ampex system to remove amplitude modulation from the modulated wave reproduced from tape before demodulating. Hopefully, this removes the amplitude fluctuations of the recovered wave before demodulation in the amplitude sensitive demodulator. This limiting action works to minimize the reproduced amplitude loss of the FM wave due to head effects. By so doing the true zero crossings of the FM wave are reproduced far more faithfully than without limiting.

It must be understood that the rate of change of these zero crossings establishes the instantaneous frequency of the system, a function we shall investigate more thoroughly in section 5.

The amplitude losses due to playback considerations have the unhappy effect of reducing the head output response to zero around eight megacycles per second. To improve bandwidth the reproduce head is caused to resonate at, typically, seven megacycles per second, extending the effective bandwidth to, say, 11 megacycles per second. (These values of frequency are only illustrative but indicate the order of magnitude).

Now by introducing the resonance within the spectrum of the FM wave, it would seem intuitive that the classical problem of tuned

circuit distortion on an impressed FM wave has been created. Indeed, this is the case. The problem is compounded, however, in that the modulating frequency is from ten cycles per second to four megacycles per second while the center (or carrier) frequency is around 6.5 megacycles per second. Worse, the deviation currently used is .5 megacycles per second.

No analysis of such an FM system is available in the literature. Personnel at Ampex Corporation have evolved the current system through five years of experience with video recording. One would suspect that by trial, error and educated guessing their current system is near the optimum for carrier frequency, deviation, playback head Q and playback head resonance.

The remainder of this paper will concern itself with the general development of a mathematical expression for the distortion caused by the tuned head circuit on the impressed FM wave. By attaining greater insight into the reasons for this distortion, guides for improved performance may be had in order to yield a better demodulated signal-to-noise ratio.

This analysis shall neglect the loss of head response, tacitly assuming that it is adequate. A clear evaluation of its effect on signal distortion may readily be had as an extension to this paper. Simila ly, the effects of eqι alization of the modulated wave may be studied. Suggestions for these investigations will be given later.

Mention must be given here to the record head resonance. While there is such a resonance and while it usually occurs near, but below, the playback head resonance its distortion effect is secondary to the playback because its resonant Q is damped to about one.

This gives a more or less linear phase shift over the frequency spectrum of interest. The full significance of this is clearer in section 5, but, in general, such a linear phase shift yields a lower order distortion to an FM wave than a high Q circuit.

Summarizing the results of this section it is seen that the record/reproduce process in wide-band video recording may be approximated by a tuned circuit. The distortion caused by this circuit on an FM wave will now be investigated.

6. Dynamic analysis of video head distortion.

Consider a linear passive network that is characterized by its unit impulse response h(t). Let this system be excited by

$$s(t) = e^{j[\Omega_c t + \theta(t)]} \qquad (3)$$

where $\theta(t)$ = some arbitrary function of time

Ω_c = carrier frequency

Define the instantaneous frequency of the excitation as

$$\Omega_i(t) = \Omega_c + \frac{d\theta}{dt} \qquad (4)$$

then (3) is

$$s(t) = e^{j\int^{t} \Omega_i(t)\,dt} \qquad (5)$$

The response of the network to this excitation, after the initial transients have died out, has been shown by E. J. Baghdady[14] to be

$$e(t) = E(t)\, e^{j[\Omega_c t + \theta(t)]} \qquad (6)$$

where E(t) is given as

$$E(t) = \sum_{n=0}^{\infty} \frac{1}{n!} C_n(t) Z^{(n)}(j\Omega_i) \qquad (7)$$

$Z(j\,\Omega_i)$ is the system function and $Z^{(n)}(j\,\Omega_i) = \frac{d^n[Z(j\Omega_i)]}{d(j\Omega_i)^n}$. $\frac{1}{n!} C_n(t)$ is the sum of all possible product terms

[14]E. J. Baghdady, Theory of Low-Distortion Reproduction of FM Signals in Linear Systems, IRE Trans. on Circuit Theory, CT-5, 3, pp. 202-214, Sept., 1958

$$\prod_{p,q,m} \left[j \frac{\theta^{(p)}}{p!} \right] \cdot \frac{1}{m!} \left[j \frac{\theta^{(q)}}{q!} \right]^m \tag{8}$$

that satisfy the requirement

$$\sum_p p + \sum_{m,q} mq = n \,, \qquad m = 0, 1, 2, 3, \cdots \tag{9}$$

p and q are positive integers different from unity, indicating p and q differentiations with respect to the independent variable t.

Baghdady has tabulated all terms through n=8 but this table will not be reproduced here as it has no immediate application.

Let us recall that $\theta(t)$ is any arbitrary function of time. This generality is not needed in this analysis and simplifications are inherent in assuming a periodic modulating wave. Consequently, let us assume such a wave, $s(\omega t)$ where ω is the fundamental repetition frequency in radians per second. Equation (7) then takes the new form

$$E_{(t)} = E(\omega t) = \sum_{n=0}^{\infty} G_n(\omega t)\, \omega^n \tag{10}$$

This we recognize as a power series expanded about $\omega = 0$. $G_n(\omega t)$ is the sum of coefficients of ω^n in the expansion of equation (7) after the substitution of $\theta^{(n)}_{(t)} = \omega^{n-1} s^{(n-1)}(\omega t)$.

Baghdady has made this substitution and tabulated the results. This is given in Table 2. Interestingly enough, this is the same expansion as that used by F. L. H. M. Stumpers[15] and B. Van der Pol[16] in previous works. As noted by B. Van der Pol and clarified by

[15] F. L. H. M. Stumpers, loc. cit.

[16] B. Van der Pol, loc. cit.

TABLE 2

Coefficients, $G_n(\omega t)$

n	$G_n(\omega t)$
0	$Z(j\Omega_i)$
1	$j\frac{1}{2!}\,S'(\omega t)\,Z^{(2)}(j\Omega_i)$
2	$j\frac{1}{3!}\,S''(\omega t)\,Z^{(3)}(j\Omega_i) + \frac{1}{2!}\left[j\frac{S'(\omega t)}{2!}\right]^2 Z^{(4)}(j\Omega_i)$
3	$j\frac{S'''(\omega t)}{4!}Z^{(4)}(j\Omega_i) + \left[j\frac{S'(\omega t)}{2!}\right]\left[j\frac{S''(\omega t)}{3!}\right]Z^{(5)}(j\Omega_i) + \frac{1}{3!}\left[j\frac{S'(\omega t)}{2!}\right]^3 Z^{(6)}(j\Omega_i)$
4	$j\frac{S^{IV}(\omega t)}{5!}Z^{(5)}(j\Omega_i) + \left\{\left[j\frac{S'(\omega t)}{2!}\right]\left[j\frac{S'''(\omega t)}{4!}\right] + \frac{1}{2!}\left[j\frac{S''(\omega t)}{3!}\right]^2\right\}Z^{(6)}(j\Omega_i) + \frac{1}{2!}\left[j\frac{S'(\omega t)}{2!}\right]^2\left[j\frac{S''(\omega t)}{3!}\right]Z^{(7)}(j\Omega_i) + \frac{1}{4!}\left[j\frac{S'(\omega t)}{2!}\right]^4 Z^{(8)}(j\Omega_i)$
	$G_n(\omega t) = \sum$ coefficients of ω^n

Baghdady, this expansion has the peculiar property that its terms diminish in magnitude with increasing n, until a minimum is reached, then increase without limits. This denotes that the expansion is not absolutely convergent but it is uniformly convergent.

An asymptotic series has the property that the error resulting from approximating with the first n terms is less than the (n+1) term. The best approximation, then, is when the (n+1) term is the smallest in the series. It is probable that such a series will yield a better approximation for a given number of terms than a convergent series.

Returning to equation (10) and Table 2, we see that, in general,

$$E(\omega t) = E_r + jE_i \tag{11}$$

Let us now assume that the video head impedance is given by $Z(j\Omega_i)$. Further, let us express

$$Z(j\Omega_i) = A(\Omega)\, e^{j\phi(\Omega_i)} \tag{12}$$

Substitution of (12) in Table 2 will yield E_r and E_i as a function of head impedance amplitude, $A(\Omega)$, of phase, $\phi(\Omega)$, and their derivatives.

Now since $s(\omega t)$ is a periodic wave we may take

$$S(\omega t) = \Delta\Omega \sin \omega t$$

For ease of notation let us make the substitutions

$$B = \frac{1}{2!}\frac{d[S(\omega t)]}{dt} = \frac{1}{2!}\,\omega\,\Delta\Omega \cos \omega t$$

$$C = \frac{1}{3!}\frac{d^2[S(\omega t)]}{dt^2} = -\frac{\omega^2}{3!}\,\Delta\Omega \sin \omega t$$

$$D = \frac{1}{4!}\frac{d^3[S(\omega t)]}{dt^3} = -\frac{\omega^3}{4!}\,\Delta\Omega \cos \omega t$$

By performing the indicated derivatives in Table 2, making the above substitutions and gathering terms, E_r has been tabulated in Table 3 and E_i, in Table 4 through n=3.

We may now rewrite equation (6) as

$$e_{(t)} = (E_r + jE_i)\, e^{j\phi(\Omega_i)}\, e^{j[\Omega_c t + \theta(t)]}$$

$$= E_r\, e^{j\tan^{-1}\frac{E_i}{E_r}}\, e^{j\phi(\Omega_i)}\, e^{j[\Omega_c t + \theta(t)]} \tag{13}$$

Since e(t) represents an FM wave, the demodulation is concerned only with the phase and not the amplitude of the response. Consequently, the instantaneous frequency of the response can be given as

$$\Omega_{i_0} = \Omega_c + \Delta\Omega \sin \omega t + \omega \Delta\Omega \cos \omega t \left\{ \phi' + \frac{E_r \frac{dE_i}{d\Omega_i} - E_i \frac{dE_r}{d\Omega_i}}{E_r^2 + E_i^2} \right\} \tag{14}$$

Where the prime denotes, as in Tables 3 and 4, derivatives with respect to Ω_i.

Tables 5 and 6 give $\frac{dE_r}{d\Omega_i}$ and $\frac{dE_i}{d\Omega_i}$, respectively.

We are now armed with some mighty tools for the analysis of tuned circuit distortion on any FM wave where the circuit impedance can be expressed as $A(\Omega)\, e^{j\phi(\Omega)}$.

To gain an insight into this problem let us return to the simple case where the n=1 term is the error term. This permits us to write

$$E_{(\omega t)} = A + j^0 \tag{15}$$

Substituting (15) into (14) gives

$$\Omega_{i_0} = \Omega_c + \Delta\Omega \sin \omega t + (\omega \Delta\Omega \cos \omega t)\, \phi' \tag{16}$$

TABLE 3

Real Components of $E(\omega t)$

n	E_r
0	A
1	$[B][A\phi'' + 2A'\phi']$
2	$[C][3A\phi''\phi' + 3A'\phi'^2 - A'''] + \frac{1}{2!}[B^2][4A\phi'''\phi' +$
	$3A\phi''^2 - A^{iv} + 6A''\phi'^2 + 12A'\phi''\phi' - A\phi'^4]$
3	$[D][4A'\phi'^3 + 6A\phi'^2\phi'' - 4A'''\phi' - 4A'\phi''' - 6A''\phi'' - A\phi^{iv}]$
	$+ [B][C][30A'\phi''\phi'^2 - A\phi'^5 + 10A''\phi'^3 - 5A^{iv}\phi' + 15A\phi''^2\phi'$
	$+ 10A\phi'^2\phi''' - 10A'''\phi'' - 10A''\phi'' - 5A'\phi^{iv} - A\phi^{v}] +$
	$\frac{1}{3!}[B^3][20A'''\phi'^3 - 6A'\phi'^5 - 15A\phi'^4\phi'' + 60A'\phi'''\phi'^2 +$
	$90A''\phi''\phi'^2 + 15A\phi^{iv}\phi'^2 + 90A'\phi''^2\phi' + 60A\phi'''\phi''\phi'$
	$- 6A^{v}\phi' - 15A^{iv}\phi'' + 15A\phi''^3 - 20A'''\phi''' - 15A''\phi^{iv}$
	$- 6A'\phi^{v} - A\phi^{vi}]$

TABLE 4

Imaginary Components of $E(\omega t)$

n	E_i
0	0
1	$[B]\,[A\phi'^2 - A'']$
2	$[C]\,[A\phi'^3 - A\phi''' - 3A'\phi'' - 3A''\phi'] + \frac{1}{2!}[B^2]\,[4A'\phi'^3$
	$+ 6A\phi'^2\phi'' - 4A'''\phi' - 4A'\phi''' - 6A''\phi'' - A\phi^{iv}]$
3	$[D]\,[A\phi'^4 - 12A'\phi''\phi' - 6A''\phi'^2 + A^{iv} - 3A\phi''^2 - 4A\phi'\phi''']$
	$+[B][C]\,[5A'\phi'^4 - 10A'''\phi'^2 + 10A\phi'^3\phi'' - 20A'\phi'''\phi'$
	$-30A''\phi''\phi' - 5A\phi^{iv}\phi' - 15A'\phi''^2 - 10A\phi''\phi''' + A^{v}]$
	$+\frac{1}{3!}\,[B^3]\,[60A'\phi''\phi'^3 - A\phi'^6 + 15A''\phi'^4 - 15A^{iv}\phi'^2 +$
	$45A\phi''^2\phi'^2 + 20A\phi'^3\phi''' - 60A'''\phi''\phi' - 60A''\phi'''\phi'$
	$-30A'\phi^{iv}\phi' - 6A\phi^{v}\phi' - 60A'\phi'''\phi'' - 45A''\phi''^2$
	$-15A\phi^{iv}\phi'' - 10A\phi'''^3 + A^{vi}]$

TABLE 5

Real Components of Derivatives of $E(\omega t)$

n	$\frac{d\,E_r}{d\,\Omega_i}$
0	A'
1	$[B][A\phi''' + 2A''\phi' + 3A'\phi'']$
2	$[C][9A'\phi''\phi' + 3A\phi'''\phi' + 3A\phi''^{2} + 3A''\phi'^{2} - A^{iv}]$
	$+ \frac{1}{2!}[B^2][16A'\phi'''\phi' + 4A\phi^{iv}\phi' + 10A\phi'''\phi'' + 15A'\phi''^{2}$
	$- A^{v} + 6A'''\phi'^{2} + 24A''\phi''\phi' - A'\phi'^{4} - 4A\phi'^{3}\phi'']$

TABLE 6

Imaginary Components of Derivatives of $E(\omega t)$

n	$\frac{dE_i}{d\Omega_i}$
0	0
1	$[B]\left[A'\phi'^2 + 2A\phi''\phi' - A'''\right]$
2	$[C]\left[A'\phi'^3 + 3A\phi''\phi'^2 - 4A'\phi''' - A\phi^{iv} - 6A''\phi'' - 3A'''\phi'\right]$ $+ \frac{1}{2!}[B^2]\left[4A''\phi'^3 + 18A'\phi''\phi'^2 + 12A\phi''^2\phi' + 6A\phi'''\phi'^2 - 4A^{iv}\phi' - 10A'''\phi'' - 10A''\phi''' - 5A'\phi^{iv} - A\phi^{v}\right]$

This is a quasi-stationary solution and represents a limiting case. It means that the circuit is completely capable of following through stationary states the variable frequency of the exciting wave. The circuit will, however, even under quasi-stationary response yield some distortion into the instantaneous frequency waveform. Clearly, this distortion is a function only of the phase characteristic and the modulating wave.

Here, then, we have our first insight into FM distortion due to a resonant circuit. But is only the n=0 term of the expansion of $E(\omega t)$ always the best solution? Obviously not. Since both ϕ and A are functions of frequency the relationship of these values and their derivatives must be computed for each discrete frequency selected. The values of successive terms in the $E(\omega t)$ expansion must be inspected to seek the minimum value, which is the error term, say, (n+1). This $(n+1)^{th}$ term represents the bound on the error for a discrete frequency but will not necessarily be the same term at another frequency. Any computational scheme must include a means for finding the minimum term or at least computing the magnitude of the term selected as the error term.

After selecting the $(n+1)^{th}$ term, the n^{th} term becomes the last term considered in the computation of distortion as given by

$$\frac{\Omega_{i_o} - \Omega_i}{\Omega_i} \times 100\% = \text{Percent distortion} \qquad (17)$$

Substitution of equations (4) and (14) into (17) and rearrangement of terms gives

$$\frac{\omega \Delta \Omega \cos \omega t \left\{ \phi' + \dfrac{E_r \dfrac{dE_i}{d\Omega_i} - E_i \dfrac{dE_r}{d\Omega_i}}{E_r^2 + E_i^2} \right\}}{\Omega_c + \Delta\Omega \sin \omega t} \times 100\%$$

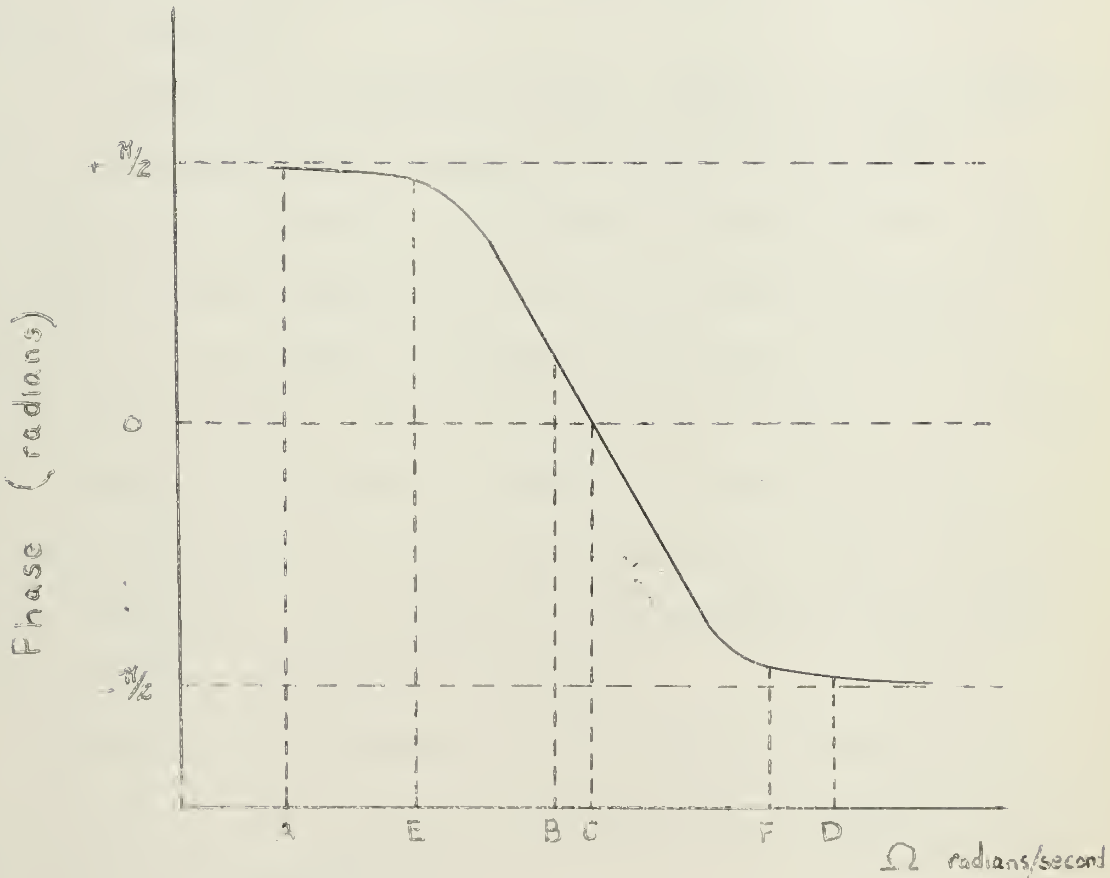
$$= \text{Percent distortion} \qquad (18)$$

Consider again the quasi-stationary solution as given in equation (16). In the light of equation (18) we see that ω, $\Delta\Omega$, ϕ' and Ω_c all are factors to be considered in obtaining distortion. For this limiting case let us further consider a typical phase response characteristic as given in Figure 1. "A" represents the low frequency extreme of frequency deviation, "B" is the center of carrier frequency, "C" is the resonant frequency, and "D" is the upper frequency extreme of deviation.

By inspection of Figure 1 and equation (18) we see that the regions of $\overline{AE}$ and $\overline{FD}$ contribute little to distortion since ϕ' in this region is very small. However, the significent distortion occurs in $\overline{EB}$ and $\overline{BF}$. If the transition from $+\pi/2$ to $-\pi/2$ radians could be made small (high Q) and the regions of $\overline{EB}$ and $\overline{BF}$ would become smaller and the sum of the distortion would be similiarly reduced. But wait; is the validity of this last sentence immediately apparent? The answer is a qualified yes, within certain error bounds and if the Q's considered are consistent with those realizable in video heads.

However, it must be understood that for the given system to be investigated a quasi-stationary solution is only the roughest of approximations. It must, therefore, be shown that the quasi-stationary solution is or is not adequate and, if not, then how many terms of the expansion in Table 2 (equation 10) must be considered.

FIGURE 1



Phase versus Angular Frequency

7. Polynomial approximation of head impedance.

It has been shown[17] that the video record/reproduce process may be approximated by a resonant circuit. Any analysis must loose its generality when tied to a specific system but, to get quantatative answers specifics must be used. Consequently, the author has selected as typical video heads those used in the Ampex Airborne Wideband Magnetic Tape Recorder manufactured under Air Force Contract Number AF33 600-37696.

Two approaches for expressing the head impedance, $Z(j\Omega_i)$, could be used: (1) Synthesizing a circuit to give the same impedance characteristics as the head, or (2) finding a polynomial which will approximate the head impedance. The latter is a much more general approach in that any degree of polynomial may be used to obtain as close a fit as desired. It is this approximation that is used in this paper. Further, the polynomial approximation technique has the advantage of permitting arbitrary impedance changes to show the effects of head equalization. This latter advantage would be of value in any extension to the investigation of this paper.

For the video heads selected a seventh-order polynomial may be found readily from the solution of eight simultaneous equations. The polynomials may be expresses as

$$A_{(\omega)} = a_7\Omega^7 + a_6\Omega^6 + a_5\Omega^5 + a_4\Omega^4 + a_3\Omega^3 + a_2\Omega^2 + a_1\Omega + a_0 \quad (19)$$

and

$$\phi_{(\omega)} = b_7\Omega^7 + b_6\Omega^6 + b_5\Omega^5 + b_4\Omega^4 + b_3\Omega^3 + b_2\Omega^2 + b_1\Omega^1 + b_0 \quad (20)$$

[17] Sec. 4

Let us consider only equation (19) since the solution of (20) is an exact parallel.

By computing the value of damped head impedance at eight discrete frequencies and substituting in equation (19), we have

$$
\begin{aligned}
A_7 &= a_7\Omega_1^7 + a_6\Omega_1^6 + a_5\Omega_1^5 + a_4\Omega_1^4 + a_3\Omega_1^3 + a_2\Omega_1^2 + a_1\Omega_1 + a_0 \\
A_6 &= a_7\Omega_2^7 + a_6\Omega_2^6 + a_5\Omega_2^5 + a_4\Omega_2^4 + a_3\Omega_2^3 + a_2\Omega_2^2 + a_1\Omega_2 + a_0 \\
A_5 &= a_7\Omega_3^7 + \cdots \qquad\qquad \cdots + a_0 \\
A_4 &= a_7\Omega_4^7 + \cdots \qquad\qquad \cdots + a_0 \\
A_3 &= a_7\Omega_5^7 + \cdots \qquad\qquad \cdots + a_0 \\
A_2 &= a_7\Omega_6^7 + \cdots \qquad\qquad \cdots + a_0 \\
A_1 &= a_7\Omega_7^7 + \cdots \qquad\qquad \cdots + a_0 \\
A_0 &= a_7\Omega_8^7 + a_6\Omega_8^6 + a_5\Omega_8^5 + a_4\Omega_8^4 + a_3\Omega_8^3 + a_2\Omega_8^2 + a_1\Omega_8 + a_0
\end{aligned}
$$

In matrix notation

$$
\begin{bmatrix} A_7 \\ A_6 \\ A_5 \\ A_4 \\ A_3 \\ A_2 \\ A_1 \\ A_0 \end{bmatrix}
=
\begin{bmatrix}
\Omega_1^7 & \Omega_1^6 & \Omega_1^5 & \Omega_1^4 & \Omega_1^3 & \Omega_1^2 & \Omega_1 & 1 \\
\Omega_2^7 & \Omega_2^6 & \Omega_2^5 & \Omega_2^4 & \Omega_2^3 & \Omega_2^2 & \Omega_2 & 1 \\
\Omega_3^7 & & & & & & & 1 \\
\Omega_4^7 & & & & & & & 1 \\
\Omega_5^7 & & & & & & & 1 \\
\Omega_6^7 & & & & & & & 1 \\
\Omega_7^7 & & & & & & & 1 \\
\Omega_8^7 & & & & & & & 1
\end{bmatrix}
\begin{bmatrix} a_7 \\ a_6 \\ a_5 \\ a_4 \\ a_3 \\ a_2 \\ a_1 \\ a_0 \end{bmatrix}
$$

or

$$
[A] = [\Omega][a] \tag{21}
$$

The matrix of unknown coefficients, $[a]$ may readily be found by multiplying each side of (21) by $[\Omega]^{-1}$. This gives

$$[\Omega]^{-1}[A] = [\Omega]^{-1}[\Omega][a]$$

but $[\Omega]^{-1}[\Omega]$ is the identity matix, hence

$$[\Omega]^{-1}[A] = [a] \tag{22}$$

$[\Omega]^{-1}$ is a matrix which may be found from the $[\Omega]$ matrix of given discrete values. Similiarly, $[A]$ is a matrix of known impedance magnitude at discrete frequencies. Consequently, as shown in (22), the unknown coefficients may be computed. Unhappily, the inversion of an eight by eight matrix is a laborious task made more difficult by the dynamic range of Ω. This task is best suited to an automatic computer.

The values of undamped head reactance, ΩL_p, and resistance, R_p, are given in Figure 2. These plotted values represent the average of 16 heads measured on a Boonton RX Meter, Type 250-A.

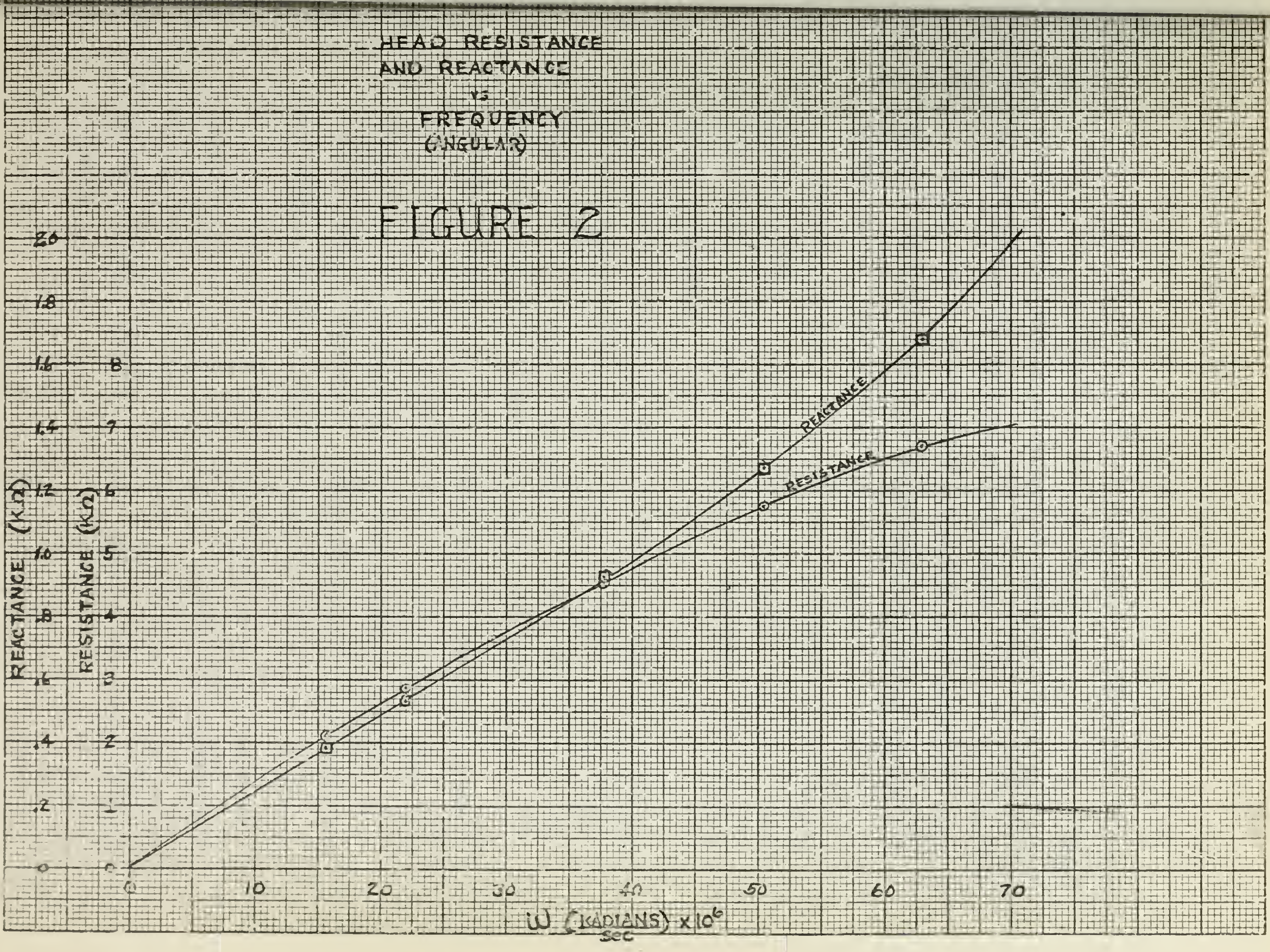

HEAD RESISTANCE
AND REACTANCE
vs
FREQUENCY
(ANGULAR)
REACTANCE
RESISTANCE
REACTANCE (KΩ)
RESISTANCE (KΩ)
ω (RADIANS/sec) x 10^6
0
10
20
30
40
50
60
70
.2
.4
.6
.8
1.0
1.2
1.4
1.6
1.8
2.0
1
2
3
4
5
6
7
8

FIGURE 2

8. Computational technique and system variables.

The question might now be raised as to what parameters are to be varied in our quest for an optimized, minimum-distortion, video head circuit? As previously seen, the parameters Ω_c, $\Delta\Omega$, ω, Ω_{res}, and Q all affect distortion directly or indirectly. These then should be the variables for any complete evaluation.

Ω_{res} and Q determine the polynomials $A(\omega)$ and $\phi(\omega)$. For these we may arbitrarily take values applicable to the scope of this investigation. These values are given in Table 7.

Substituting $A(\omega)$, $\phi(\omega)$ and their derivatives as well as Ω_c, $\Delta\Omega$ and ω into equation (18) will yield percent distortion. The multiplicity of parameters to be varied again suggests the use of an automatic computer.

As seen in section 7, the inversion of an eight-by-eight matrix is necessitated in this analysis. Careful selection of the frequencies to be used can minimize the computational problem. The author has selected frequencies to require only three matrix inversions. However, for the frequencies used, the amplitude and phase of the circuit is needed. Tables 8 through 12 give these impedance values versus frequency. Three decimals are carried in order to minimize the effects of round-off errors during computation. The reader's attention is invited to the tabulated values of Ω. It can be seen that only three sets are given reflecting the three matrix inversions previously mentioned. In general Ω ranges beyond the region of interest but this aids in insuring good behavior of the approximating polynomial over the frequency band required in this distortion analysis.

TABLE 7

Selected Ω_{res} and Q Values

$\Omega_{res} \times 10^6 \frac{\text{radians}}{\text{sec}}$	Q	$\Omega_{res} \times 10^6 \frac{\text{radians}}{\text{sec}}$	Q
3.4	4.5	4.6	4.5
	4.0		4.0
	3.0		3.0
	2.0		2.0
	1.0		1.0
3.8	4.5	5.0	4.5
	4.0		4.0
	3.0		3.0
	2.0		2.0
	1.0		1.0
4.2	4.5	5.4	4.0
	4.0		3.0
	3.0		2.0
	2.0		1.0
	1.0		

A more exact fit of the approximating polynomial may be had by selecting values of Ω closer together, by going to a higher order polynomial, or both. The higher order polynomial does not seem a sound intuitive procedure in that the impedance function is quite well behaved over the entire frequency range and even a seventh order polynomial is going somewhat beyond the practical.

Implicit in all the previous discussion about the behavior of the approximating polynomial is the equally good behavior of all its derivatives. With a lower order approximating polynomial a smoothing function would probably be required for meaningful results. This may or may not be true of the seventh order polynomial. Evaluation of this can only be made during actual computation of distortion.

The dynamic range of values in the eight-by-eight frequency matrix to be inverted is about 10^6. Care must be exercised to preserve this accuracy during inversion or the approximating polynomial will yield excessive errors.

In review let us list a sequential computational technique.

1. Select from Table 8 through 12 the applicable impedance values and frequencies.

2. Find the inverse of the selected frequency matrix.

3. Perform multiplication of the matrix on the impedance values to yield $A(\omega)$ and $\phi(\omega)$.

4. Select Ω_c, $\Delta\Omega$, Ω_{res}, and ω. (Probably a range of values for each).

5. Substitute values into equation (18) to find distortion for each discrete combination of parameters selected. The convergence of

$E(\omega t)$ must be established as discussed in section 8.

6. Note that as t varies such that the sine and cosine complete one cycle, the distortion too has completed one cycle and will now repeat.

7. Proceed to the next set of parameters and repeat computations again allowing the sine and cosine to range over one cycle.

The cyclic phenomenon of distortion is a little recognized facet of the effects of a tuned circuit on an FM wave. Presumably these effects have been previously ignored for in the usual case of FM transmission Ω_c is large compared to all other parameters and, hence, from equation (18), distortion is small.

Tables 8 through 12 gives impedance values for a given video head. There is no generality lost in this analysis if another video head of different characteristics is substituted. All that is then required is the new set of impedance values for the given discrete frequencies.

Moreover, a fictitious video head or one with "equalization" effects imposed may be investigated by again substituting the impedance values corresponding to the discrete frequency. This technique will bring the effects of equalization on signal-to-noise ratio sharply into view and give a firm foundation for laboratory work which has been to date quite empirical.

TABLE 8

Impedence Values for $Q = 4.5$

Ω_{res} $\times 10^6$ $\frac{\text{radians}}{\text{sec}}$	Ω $\times 10^6$ $\frac{\text{radians}}{\text{sec}}$	ϕ radians	A $\times 10^3$ ohms	Ω_{res} $\times 10^6$ $\frac{\text{radians}}{\text{sec}}$	Ω $\times 10^6$ $\frac{\text{radians}}{\text{sec}}$	ϕ radians	A $\times 10^3$ ohms
34	22	1.236	.870	46	30	1.240	1.185
	26	1.103	1.360		34	1.155	1.628
	30	.796	2.372		38	1.003	2.367
	34	.000	3.72		42	.688	3.689
	38	- .831	2.714		46	.000	5.13
	42	-1.156	1.751		50	- .714	4.103
	46	-1.295	1.262		54	-1.054	2.848
	50	-1.366	.989		58	-1.214	2.110
38	22	1.281	.773	50	38	1.141	1.903
	26	1.204	1.112		42	.989	2.724
	30	1.064	1.694		46	.667	4.201
	34	.752	2.804		50	.000	5.67
	38	.000	4.16		54	- .673	4.712
	42	- .786	3.186		58	-1.023	3.302
	46	-1.123	2.088		62	-1.190	2.449
	50	-1.267	1.518		66	-1.282	1.944
42	30	1.177	1.368				
	34	1.029	2.024				
	38	.709	3.243				
	42	.000	4.63				
	46	- .750	3.628				
	50	-1.088	2.436				
	54	-1.240	1.800				
	58	-1.323	1.421				

TABLE 9

Impedance Values for Q = 4.0

Ω_{res} $\times 10^6 \frac{radians}{sec}$	Ω $\times 10^6 \frac{radians}{sec}$	ϕ radians	A $\times 10^3$ ohms	Ω_{res} $\times 10^6 \frac{radians}{sec}$	Ω $\times 10^6 \frac{radians}{sec}$	ϕ radians	A $\times 10^3$ ohms
34	22	1.210	.861	46	30	1.213	1.174
	26	1.063	1.332		34	1.119	1.601
	30	.741	2.241		38	.955	2.291
	34	.000	3.31		42	.635	3.446
	38	- .767	2.550		46	.000	4.56
	42	-1.103	1.708		50	- .654	3.812
	46	-1.255	1.247		54	- .995	2.748
	50	-1.334	.982		58	-1.166	2.070
38	22	1.260	.768	50	38	1.103	1.868
	26	1.174	1.098		42	.941	2.640
	30	1.021	1.651		46	.613	3.907
	34	.698	2.638		50	.000	5.04
	38	.000	3.70		54	- .613	4.349
	42	- .723	2.979		58	- .963	3.175
	46	-1.068	2.029		62	-1.141	2.398
	50	-1.225	1.497		66	-1.242	1.919
42	30	1.143	1.346	54	38	1.181	1.621
	34	.984	1.967		42	1.083	2.167
	38	.656	3.038		46	.909	3.036
	42	.000	4.12		50	.569	4.397
	46	- .687	3.376		54	.000	5.52
	50	-1.031	2.359		58	- .602	4.759
	54	-1.195	1.770		62	- .939	3.539
	58	-1.286	1.407		66	-1.117	2.716

TABLE 10

Impedance Values for Q =3.0

Ω_{res} $\times 10^6$ radians/sec	Ω $\times 10^6$ radians/sec	ϕ radians	A $\times 10^3$ ohms	Ω_{res} $\times 10^6$ radians/sec	Ω $\times 10^6$ radians/sec	ϕ radians	A $\times 10^3$ ohms
34	22	1.133	.834	46	30	1.135	1.136
	26	.954	1.243		34	1.020	1.517
	30	.612	1.907		38	.832	2.075
	34	.000	2.48		42	.511	2.841
	38	- .619	2.132		46	.000	3.42
	42	- .961	1.568		50	- .515	3.086
	46	-1.141	1.193		54	- .845	2.450
	50	-1.241	.956		58	-1.036	1.938
38	22	1.196	.751	50	38	.999	1.760
	26	1.086	1.054		42	.814	2.375
	30	.904	1.522		46	.491	3.202
	34	.570	2.215		50	.000	3.78
	38	.000	2.78		54	- .480	3.491
	42	- .577	2.456		58	- .811	2.804
	46	- .921	1.844		62	-1.007	2.230
	50	-1.104	1.421		66	-1.127	1.831
42	30	1.048	1.283	54	38	1.094	1.557
	34	.863	1.795		42	.974	2.029
	38	.530	2.518		46	.780	2.706
	42	.000	3.09		50	.453	3.572
	46	- .545	2.759		54	.000	4.14
	50	- .882	2.123		58	- .471	3.814
	54	-1.069	1.668		62	- .787	3.105
	58	-1.181	1.356		66	- .980	2.510

TABLE 11

Ω_{res} $\times 10^6 \frac{\text{radians}}{\text{sec}}$	Ω $\times 10^6 \frac{\text{radians}}{\text{sec}}$	ϕ radians	A $\times 10^3$ ohms	Ω_{res} $\times 10^6 \frac{\text{radians}}{\text{sec}}$	Ω $\times 10^6 \frac{\text{radians}}{\text{sec}}$	ϕ radians	A $\times 10^3$ ohms
34	22	.989	.769	46	30	.995	1.051
	26	.776	1.067		34	.851	1.338
	30	.445	1.429		38	.662	1.725
	34	.000	1.65		42	.363	2.063
	38	- .436	1.552		46	.000	2.28
	42	- .745	1.297		50	- .357	2.190
	46	- .944	1.063		54	- .632	1.935
	50	-1.078	.890		58	- .825	1.654
38	22	1.077	.711	50	38	.826	1.539
	26	.931	.955		42	.630	1.925
	30	.720	1.277		46	.346	2.303
	34	.409	1.633		50	.000	2.52
	38	.000	1.85		54	- .330	2.449
	42	- .403	1.766		58	- .599	2.181
	46	- .705	1.501		62	- .795	1.883
	50	- .901	1.247		66	- .932	1.628
42	30	.888	1.149	54	38	.940	1.415
	34	.677	1.480		42	.797	1.755
	38	.378	1.838		46	.596	2.160
	42	.000	2.06		50	.317	2.544
	46	- .379	1.970		54	.000	2.76
	50	- .668	1.703		58	- .324	2.676
	54	- .862	1.445		62	- .580	2.403
	58	- .995	1.230		66	- .768	2.099

TABLE 12

Impedance Values for Q = 1

Ω_{res} 10^6 radians/sec	Ω 10^6 radians/sec	ϕ radians	A 10^3 ohms	Ω_{res} 10^6 radians/sec	Ω 10^6 radians/sec	ϕ radians	A 10^3 ohms
34	22	.689	.586	46	30	.696	.803
	26	.476	.698		34	.545	.923
	30	.239	.786		38	.373	1.023
	34	.000	.827		42	.190	1.098
	38	- .225	.820		46	.000	1.14
	42	- .419	.778		50	- .183	1.140
	46	- .583	.722		54	- .343	1.102
	50	- .712	.660		58	- .483	1.046
38	22	.800	.579	50	38	.521	1.042
	26	.625	.697		42	.357	1.142
	30	.431	.809		46	.181	1.222
	34	.218	.888		50	.000	1.26
	38	.000	.93		54	- .168	1.260
	42	- .206	.921		58	- .322	1.224
	46	- .389	.878		62	- .458	1.167
	50	- .543	.822		66	- .573	1.100
42	30	.580	.812	54	38	.635	1.039
	34	.397	.913		42	.494	1.163
	38	.200	.990		46	.335	1.265
	42	.000	1.03		50	.165	1.340
	46	- .195	1.03		54	.000	1.38
	50	- .368	.989		58	- .165	1.380
	54	- .512	.932		62	- .311	1.342
	58	- .635	.870		66	- .439	1.284

9. Conclusions.

The development of this paper has been a systematic reduction of the difficult problem of video head distortion. It would now be wise to review the steps which have led to the conclusions given in the previous sections.

1. The classical FM wave approach may be used over the more difficult Cambi expression.
2. The major consideration in recording/reproducing is in playback,* so consideration of the latter function only will yield, at least, first order effects.
3. The playback problem is limited to amplitude loss as a function of frequency.
4. The record/reproduce process in wide-band video recording may be approximated by a tuned circuit.

Based upon the foregoing simplifications the "dynamic" analysis technique may be applied. In doing so it has resulted in the distortion expression

$$\frac{\omega \Delta\Omega \cos\omega t \left\{ \phi' + \dfrac{E_r \dfrac{dE_i}{d\Omega_i} - E_i \dfrac{dE_r}{d\Omega_i}}{E_r^2 + E_i^2} \right\}}{\Omega_c + \Delta\Omega \sin\omega t} \times 100\% \qquad (18)$$

From this expression several conclusions have been drawn.

1. A polynomial approximation of circuit impedance characteristics, $A(\Omega)$ and $\phi(\Omega)$, gives an easier computational technique than circuit synthesis. Moreover, it more readily lends itself to arbitrary change to observe post-modulation equalization effects.
2. Ω_{res} and Q determine the polynomials $A(\Omega)$ and $\phi(\Omega)$.

Upon computing these functions, E_r, E_i and derivatives may be found and combined with the parameters Ω, $\Delta\Omega$ and ω may be substituted into equation (18) to determine the percentage distortion. Clearly, distortion is a function of these parameters and time, t.

3. From a quasi-stationary consideration a low Q circuit will yield the least distortion. But this compromises the desired signal-to-noise ratio due to loss of circuit response at resonance as well as giving a reduced bandwidth. The results given in equation (18) lead to the computational scheme given in 1 and 3, above, for determining the price one pays (in terms of distortion) for using higher Q's in the video head playback circuit.

4. A digital computor will greatly ease the computational burden of this thesis. Programming is straight-forward and the technique has been clearly indicated in the previous section. No electronic background of the programmer is required.

BIBLIOGRAPHY

1. J. R. Carson and T. C. Fry, Variable-Frequency Electric Circuit Theory, B. S. T. J., 16, pp. 513-540, Oct., 1937.

2. B. Van der Pol, The Fundamental Principles of Frequency Modulation, J. IEE (London), 93, pt. 3, pp. 153-158, May, 1946.

3. F. L. H. M. Stumpers, Distortion of Frequency Modulated Signals in Electrical Networks, Commun. News, 9, pp. 82-92, Apr., 1948.

4. C. E. Ginsburg, Description of the Ampex Videotape Recorder, J. Society of Motion Picture and Television Engineers, 66, No. 4, Apr., 1957.

5. C. E. Anderson, The Modulation System of the Ampex Videotape Recorder, J. SMPTE, 66, pp. 182-184, Apr., 1957.

6. R. M. Dolby, Rotary-Head Switching in the Ampex Videotape Recorder, J. SMPTE, 66, pp. 184-188, Apr., 1957.

7. E. Cambi, Trigonometric Components of a Frequency Modulated Wave, Proc. IRE, 36, pp. 42-49, Jan., 1948.

8. S. J. Begun, Magnetic Recording, Murray Hill Books, Inc., 1949.

9. R. L. Wallace, Reproduction of Magnetically Recorded Signals, B. S. T. J., 30, pp. 1145-1173, Oct., 1951.

10. W. K. Westmijze, Studies on Magnetic Recording, Philips Research Reports, 8, 1953.

11. A. S. Hoagland, Magnetic Drum Recording of Digital Data, AIEE Trans., 73, pp. 381-385, Sept., 1954.

12. E. D. Daniel, P. E. Axon and W. T. Frost, A Survey of Factors Limiting the Performance of Magnetic Recording Systems, Proc. IEE, 104, Part B, pp. 158-168, March, 1957

13. W. T. Frost, Improvements in the Performance of High-Frequency Magnetic Recording Heads, Ampex Tech. Rpt. 59-6, Feb., 1959.

14. E. J. Baghdady, Theory of Low-Distortion Reproduction of FM Signals in Linear Systems, IRE Trans. on Circuit Theory, CT-5, 3, pp. 202-214, Sept., 1958.

15. E. J. Baghdady, On the Response of a Linear System to a FM Signal, IRE Trans. on Circuit Theory, CT-6, 4, pp. 387-388, Dec., 1959.

16. A. Hund, Frequency Modulation, McGraw-Hill Book Company, Inc., 1942.

17. Federal Works Agency Work Projects Administration for the City of New York, Tables of Circular and Hyperbolic Sines and Cosines for Radian Arguments, 1940.

18. Work Projects Administration of the Federal Works Agency, Table of Circular and Hyperbolic Tangents and Cotangents for Radian Arguments, Columbia University Press, 1943.

19. A. S. Gladwin, The Distortion of Frequency-Modulated Waves by Transmission Networks, Proc. IRE, 35, pp. 1436-1445, Dec., 1947.

APPENDIX I[18]

The classical FM derivation starts with a reactance modulator whose tank capacitance is C in the absence of modulation and a maximum swing of ΔC with modulation.

Writing the loop equations for this tank we have

$$L \frac{di}{dt} + e_c = 0$$

We may neglect the tank losses since we are usually dealing with a high Q circuit.

From the relations for current and voltage,

$$i = dq/dt$$

$$e = q/C_t$$

we may write

$$\frac{d^2q}{dt^2} + \frac{q}{C_t L} = 0 \qquad \text{(A)}$$

but $\Omega_t = 2\pi F_t = \frac{1}{\sqrt{L C_t}}$. Substituting in (A) gives

(B)

Changing the form of equation (B) to show variations in frequency vice tank capacitance using

$$F_t = \frac{\Omega_t}{2\pi} = F + \Delta F \cos \omega t$$

[18]A. Hund, Frequency Modulation, McGraw-Hill Book Company, Inc., 1942

gives

$$\frac{d^2 q}{dt^2} + q\left(\Omega^2 + .5\Delta\Omega^2 + 2\Omega\Delta\Omega\cos\omega t + .5\Delta\Omega^2\cos 2\omega t\right) = 0 \qquad \text{(C)}$$

Equation (C) has no limitation and is valid for all F and ΔF.

Now, consider the case where

$$\Delta\Omega << \Omega$$

then (C) becomes

$$\frac{d^2 q}{dt^2} + \Omega^2\left(1 + 2\frac{\Delta F}{F}\cos\omega t\right) q = 0$$

For a solution of this equation, let $q = e^{\int x\,dt}$. Then,

$$\frac{dx}{dt} + x^2 + \Omega^2\left(1 + 2\frac{\Delta F}{F}\cos\omega t\right) = 0 \qquad \text{(D)}$$

Further assume $\Delta F/f$ is small which is so when $f << F$. The solution to (D) is then given as,

$$\left.\begin{matrix} x_1 \\ x_2 \end{matrix}\right\} = \pm j\Omega\sqrt{1 + 2\frac{\Delta F}{F}\cos\omega t}$$

and by using the first two terms of the binomial expansion and applying the above assumption we have,

$$\left.\begin{matrix} x_1 \\ x_2 \end{matrix}\right\} \doteq \pm j\left(\Omega + \Delta\Omega\cos\omega t\right)$$

Then

$$q = e^{\int x_1 dt} + e^{\int x_2 dt} = K\cos\left[\int(\Omega + \Delta\Omega\cos\omega t)\,dt\right]$$

$$= K\cos\left(\Omega t + \frac{\Delta\Omega}{\omega}\sin\omega t\right) \qquad \text{(E)}$$

which we recognize as the classical expression for an FM wave.

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